Principles of Communications
ECS 332

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

4.2 Energy and Power

Office Hours:

BKD, 6th floor of Sirindhralai building

Tuesday	9:00-10:00
Wednesday	14:20-15:20
Thursday	9:00-10:00

Inner Product (Cross Correlation)

Complex conjugate

- Vector

$$\langle \vec{x}, \vec{y} \rangle = \vec{x} \cdot \vec{y}^{*} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}^{*} = \sum_{k=1}^{n} x_k y_k^{*}$$

- Waveform: Time-Domain

$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y^{*}(t) dt$$

these give the same value (Parseval's theorem)

- Waveform: Frequency Domain

$$\langle X, Y \rangle = \int_{-\infty}^{\infty} X(f) Y^{*}(f) df$$

Orthogonality

- Two signals are said to be **orthogonal** if their **inner product** is zero.
- The symbol $\perp$ is used to denote orthogonality.

Vector:

$$\langle \vec{a}, \vec{b} \rangle = \vec{a} \cdot \vec{b}^* = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}^* = \sum_{k=1}^{n} a_k b_k^* = 0$$

Time-domain:

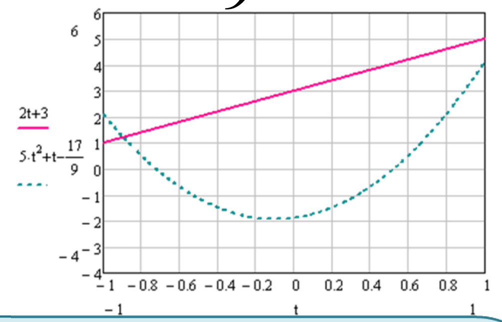
$$\langle a, b \rangle = \int_{-\infty}^{\infty} a(t) b^*(t) dt = 0$$

Frequency domain:

$$\langle A, B \rangle = \int_{-\infty}^{\infty} A(f) B^*(f) df = 0$$

Example:

$$2t + 3 \text{ and } 5t^2 + t - \frac{17}{9} \text{ on } [-1, 1]$$



Example (Fourier Series):

$$\sin\left(2\pi k_1 \frac{t}{T}\right) \text{ and } \cos\left(2\pi k_2 \frac{t}{T}\right) \text{ on } [0, T]$$

$$e^{j 2\pi n \frac{t}{T}} \text{ on } [0, T]$$

Important Properties

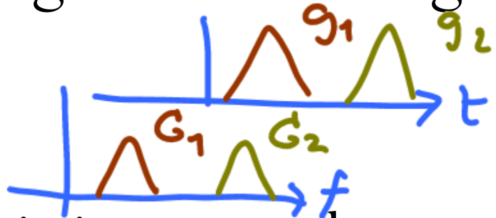
- Parseval's theorem

$$\langle x, y \rangle \equiv \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df \equiv \langle X, Y \rangle$$

It is therefore sufficient to check only on the "convenient" domain.

$$x(t) \perp y(t) \quad \text{iff} \quad X(f) \perp Y(f).$$

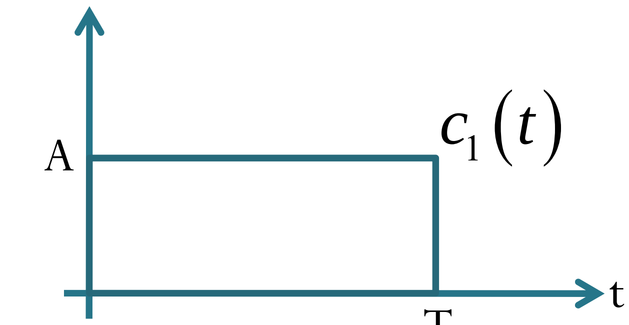
- Useful observation: If the non-zero regions of two signals
  TDMA ← - do not overlap in time domain or
  FDMA ← - do not overlap in frequency domain,

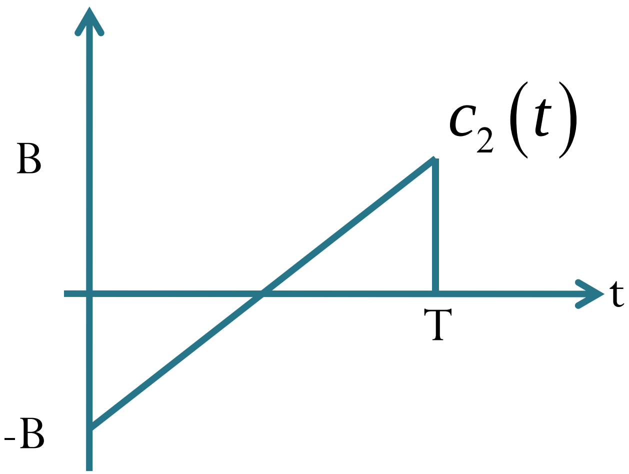


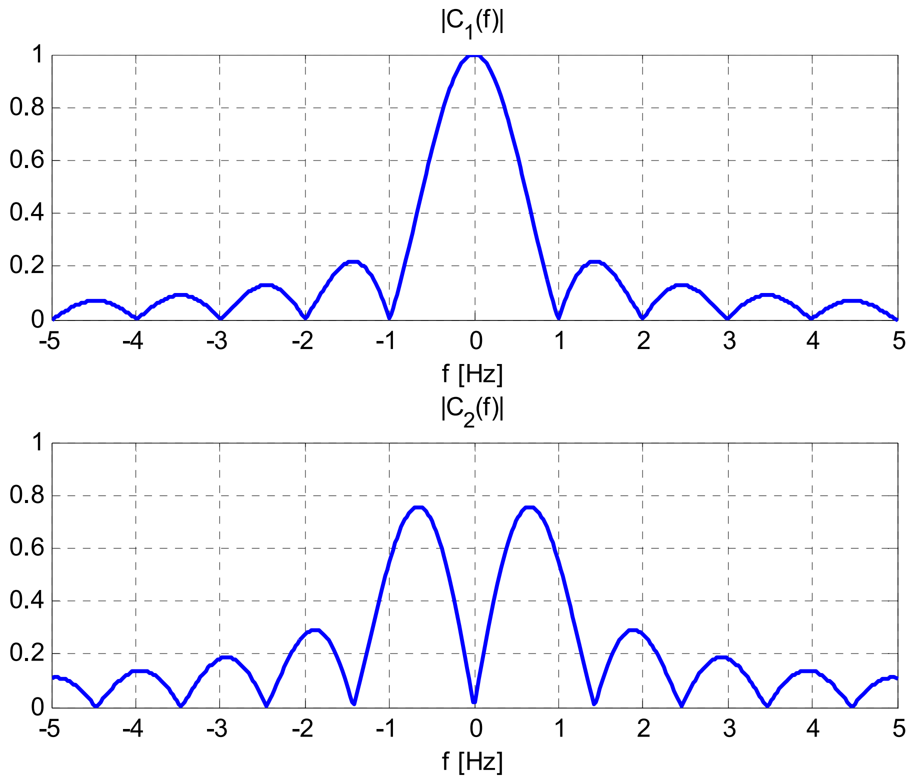
then the two signals are orthogonal (their inner product = 0).

- However, in general, orthogonal signals may overlap both in time and in frequency domain.

Orthogonality: Example 1







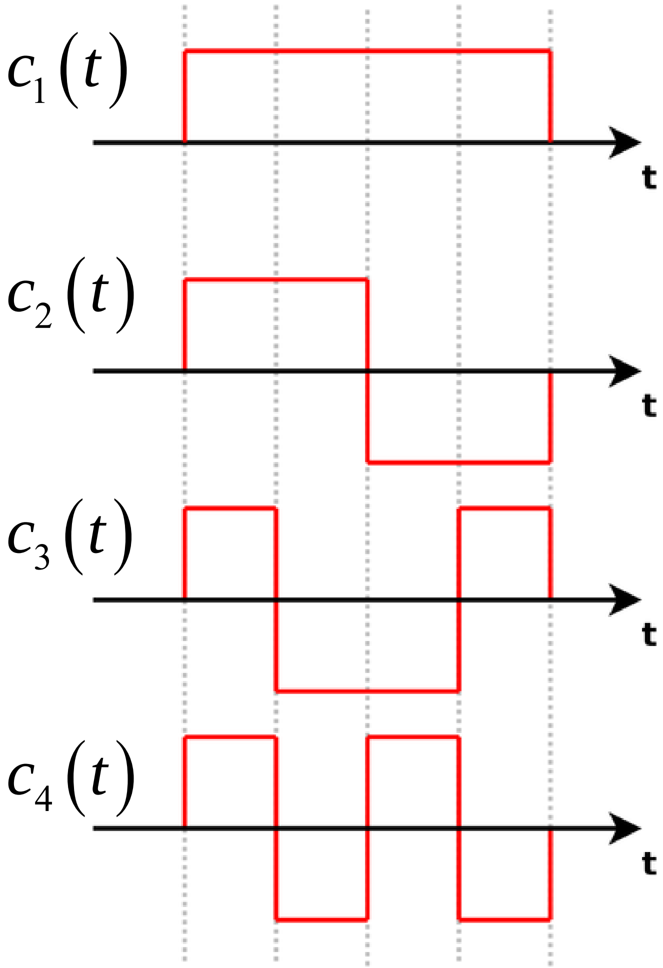
[CDMAEx.m]

The two waveforms above overlaps both in time domain and in frequency domian.

CDMA

Orthogonality: Example 2

An example of four "mutually orthogonal" signals.



When $i \neq j$,

$$\langle c_i(t), c_j(t) \rangle = 0$$